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Physics of Neutron Stars: From the Core to the Crust

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Abstract.

Recently neutron star masses have been estimated to very high degree of accuracy. The accurately measured highest neutron star mass is 2.01±0.04 $M_\odot$. This puts a strong constraint on the β-equilibrated equation of state (EoS) for neutron stars. Observed neutron star masses are also direct probes of compositions of dense matter. Novel phases of matter such as hyperons, Bose-Einstein condensates of (anti)-kaons and quarks might exist in neutron star interior. The theoretical mass-radius relationship of neutron stars including exotic forms of matter is calculated and it is shown to be compatible with massive neutron stars. Unlike neutron star masses, measurements of their radii are not accurate. The moments of inertia of pulsars could be used as a tool for the determination of neutron star radii.

On the other hand, the compositions and EoS of magnetized neutron star crusts might be probed using asteroseismology. The role of magnetized crusts on quasi periodic oscillations (QPOs) in giant flares of magnetars identified as torsional shear modes is discussed in this article.

Keywords : equation of state - magnetic fields - stars:neutron

1. Introduction

Observations on neutron stars could provide important inputs in understanding the compositions and EoS from the crust to the core. Neutron star masses have been estimated to very high degree of accuracy (Weisberg and Taylor 2005; Lattimer and Prakash 2007). This has been possible because post-Keplerian parameters such as orbital decay, periastron advance, Shapiro delay, time dilation have been measured in many pulsars. It is worth mentioning here that an accuracy of 0.0002 $M_\odot$ was reported in the mass measurements of binary pulsar PSR1913+16 (Weisberg and Taylor

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2005). Currently the accurately measured highest neutron star mass is 2.01±0.04 $M_\odot$ (Antoniadis et al. 2013). This puts a strong constraint on the β-equilibrated EoS.

It has been long debated whether exotic forms of matter such as hyperons, Bose-Einstein condensates of (anti)-kaons and quarks may exist in neutron star interior or not. As the baryon density reaches a few times normal nuclear matter density in neutron star interior, those novel phases of strange matter might appear there. It is to be noted that strange matter typically makes the EoS softer resulting in a smaller maximum mass neutron star than that of the nuclear EoS (Glendenning 1997).

Soft Gamma-ray repeaters (SGRs) and Anomalous X-ray pulsars (AXPs) are very good candidates for magnetars which are neutron stars with very high surface magnetic fields $\sim 10^{15}$ G (Duncan and Thomson 1992; Duncan 1998; Kouveluotou et al. 1998). Giant flares in SGRs might be caused by the evolving magnetic field and its stress on the crust of magnetars. It was argued that starquakes associated with giant flares could excite Seismic Oscillations (Duncan 1998). Torsional shear modes of magnetars with lower excitation energies would be easily excited. In this case, oscillations are restored by the Coulomb forces of crustal ions. Quasi-periodic oscillations (QPOs) were found in the decaying tail of giant flares (Israel et al. 2005; Watts and Strohmayer 2007; Watts 2011). These findings implied that QPOs might be torsional shear mode oscillations of magnetar crusts (Duncan 1998). Frequencies of the observed QPOs ranged from 18 Hz to 1800 Hz. It may be worth noting here that torsional shear mode frequencies are sensitive to the shear modulus of neutron star crusts. Furthermore, the shear modulus strongly depends on the composition of neutron star crusts.

We organize the paper in the following way. We describe the mass-radius relationship of neutron stars in section 2. Section 3 is devoted to the calculation of torsional shear modes using magnetized crusts. Section 4 gives the summary and conclusions.

2. Mass-Radius Relationship of Neutron Stars

The theoretical mass-radius relationship of compact stars could be compared with measured masses and radii obtained from observations. This leads to constraining the composition and EoS of dense matter in neutron star interior. Masses and radii of non-rotating neutron stars are calculated theoretically using the Tolman-Oppenheimer-Volkoff (TOV) equation and an EoS as an input to the TOV equation. The EoS with hyperon matter and EOS undergoing phase transitions from nuclear matter to (anti)-kaon condensed matter are used for calculations of the mass-radius relationship. The density dependent relativistic field theoretical models where baryon-baryon interaction is mediated by the exchange of scalar and vector mesons is adopted here (Glendenning 1997). Baryon-meson couplings are dependent on baryon density. Similarly, the (anti)kaon-baryon interaction in the (anti)kaon condensed phase is treated in the same footing as the baryon-baryon interaction.

Figure 1 shows the mass-radius relationship of non-rotating neutron stars. The mass-radius curve corresponding to the β-equilibrated nuclear EoS denoted by HS(DD2) is represented by the black line. The maximum neutron star mass is 2.42 $M_\odot$ and the radius is in 11.87 km in this case (Char and Banik 2014; Banik et al. 2014). The red line in Fig. 1 corresponds to the β-equilibrated Λ hyperon EoS denoted by BHB$\Lambda\phi$. The presence of strangeness degrees of freedom such as hyperons might make the EoS softer which is incompatible with the massive neutron star in most cases. This is known as the hyperon puzzle. It has been argued that the hyperon-hyperon repulsive interaction due to the exchange of strange vector meson makes the EoS stiffer and might overcome the puzzle. It is observed in Fig. 1 that the repulsive Λ-Λ interaction mediated by ϕ mesons indeed makes the BHB$\Lambda\phi$ EoS stiffer resulting in 2.1 $M_\odot$ maximum mass neutron star corresponding to a radius 11.57 km (Char and Banik 2014; Banik et al. 2014). Char and Banik (2014) extended this calculation to include Ξ hyperons as well as (anti)-kaon condensates. For all those cases, they obtained massive neutron stars compatible with 2 $M_\odot$ (Char and Banik 2014). This shows that there is room for the presence of exotic components for example, hyperons and Bose-Einstein condensates of (anti)-kaons in neutron star interior.

Unlike masses, radii of neutron stars are not known accurately yet. Highly relativistic binary systems such as the double pulsar system PSR J00737-3039 for which masses of both pulsars are known accurately, might facilitate the precise measurement of moment of inertia (I) of one pulsar. The relativistic spin-orbit coupling could manifest in an extra advancement of periastron over and above the higher order post Newtonian corrections. The measurement of spin-orbit coupling effect results in the estimation of the moment of inertia of a pulsar in the double pulsar binary (Damour and Schaefer 1988). Consequently, this could overcome the uncertainties in the determination of radius (R) because dimensionally $I \propto MR^2$ (Lattimer and Schutz 2005). With the advancement of the Square Kilometre Array (SKA), it would be possible to determine the moment of inertia of a pulsar because of high precision timing technique in the SKA.

3. Probing Magnetar Crusts through Asteroseismology

Asteroseismology is an important tool to study neutron star crusts. Spectacular giant flares of gamma rays powered by the decay of magnetic field were observed in three SGRs (Mazets et al. 1979; Hurley et al. 1999, 2005). It is believed that SGRs and AXPs represent a new class of neutron stars with very strong surface magnetic fields ($\sim 10^{15}$ G) known as magnetars. The rapidly changing field is strong enough to generate tremendous stress leading to the breaking of magnetar crusts (Thompson and Duncan 1995). This triggers giant flares and QPOs identified as torsional shear modes of magnetar crusts (Watts and Strohmayer 2007). In the decaying tails of giant flairs in SGR 1900+14 and SGR 1806-20, frequencies of QPOs ranged from 18 Hz to 1800 Hz.

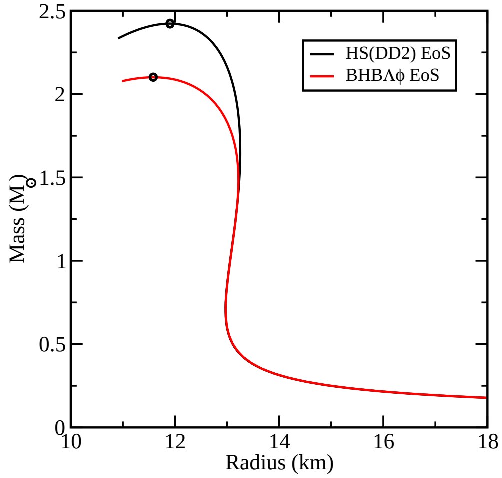

Figure 1. Mass-Radius relationship with (red line) and without (black line) Λ hyperons in the density dependent relativistic field theoretical model.

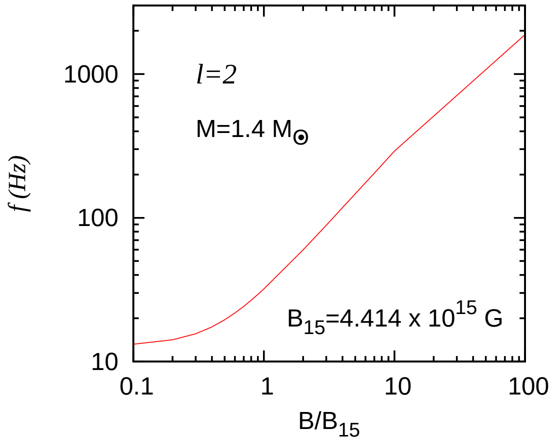

Figure 2. Fundamental shear mode frequency of a 1.4 $M_\odot$ neutron star is plotted as a function of magnetic field.

Torsional shear mode frequencies depend on the shear modulus of the magnetar crusts. On the other hand, the shear modulus is sensitive to the compositions and EoS of magnetar crusts. It is possible to extract information about the properties of magnetized crusts from observed shear mode frequencies. The ground state properties of crusts are influenced by strong magnetic fields. In particular, the Landau quantization of electrons modifies the compositions and EoS of outer and inner crusts. This effect was investigated in details using the Skyrme (SkM) nucleon-nucleon interaction (Nandi et al. 2011; Nandi and Bandyopadhyay 2013).

Torsional shear mode frequencies are calculated using magnetized crusts (Nandi

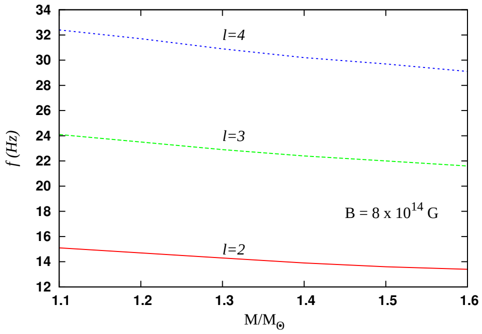

Figure 3. Torsional shear mode frequencies corresponding to $n = 0$ and $\ell = 2, 3, 4$ are plotted as a function of neutron star masses for $B = 8 \times 10^{14}$ G.

et al. 2011; Nandi and Bandyopadhyay 2013; Sotani et al. 2007). Figure 2 shows frequencies of fundamental ($n = 0$, $\ell = 2$) torsional shear modes of a 1.4 M$_\odot$ neutron star as a function of magnetic field. Here the magnetic field strength is normalized as B/B_{15} where $B_{15} = 10^2 B_c^e$ and the critical field of electrons $B_c^e = 4.414 \times 10^{13}$ G. It is found that the frequencies of the fundamental mode increase with magnetic fields. Observed frequencies of QPOs of a magnetar are explained in terms of frequencies of fundamental and overtones of torsional shear modes calculated in our model using the same composition of the magnetized crust that we developed (Nandi and Bandyopadhyay 2013). We discuss the theoretically calculated frequencies of QPOs in our model with those of SGR 1806-20. The dependence of torsional shear mode frequencies with neutron masses is shown in Fig. 3. Fundamental torsional shear mode frequencies corresponding to $n = 0$ and $\ell = 2$ and higher values of ℓ are plotted as a function of neutron star masses for magnetic field B= 8 $\times 10^{14}$. For a neutron star of 1.4 M$_\odot$, the results of our calculation is not in good agreement with the three lower frequencies (18,26 and 30 Hz) of SGR 1806-20. However, for the same mass neutron star, frequencies of overtones of shear modes in our calculation is in very good agreement with the higher torsional frequencies (93, 150, 626 and 1838 Hz) of SGR1806-20 (Nandi and Bandyopadhyay 2013).

4. Summary and Conclusions

Mass-radius relationships of massive neutron stars including hyperons and Bose-Einstein condensates of (anti)-kaons within the density dependent relativistic field theoretical models are discussed in this article. It is noted that the repulsive Λ-Λ interaction mediated by ϕ mesons makes the Λ hyperon EoS stiffer resulting in 2 M$_\odot$ neutron star. Furthermore, the observed 2 M$_\odot$ neutron star can be constructed when the EoS includes other hyperon species as well as Bose-Einstein condensates of (anti)-kaons.

Observed frequencies of QPOs in giant flares of SGRs are used to probe the compositions and EoS of magnetar crusts. The effects of magnetized crusts on the QPOs identified as torsional shear modes of crusts are discussed. It is noted that the frequencies of QPOs might constrain the crustal compositions if masses of neutron stars are known.

The high precision timing technique in the SKA would lead to the determination of moments of inertia of pulsars in highly relativistic binary systems. Consequently, the estimation of radii of pulsars within 10% accuracy might be possible (Lattimer and Schutz 2005).

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