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Estimating the mass of IGR J17091-3624: statistical challenges and methods

N. Iyer[1,2*], A. Nandi[1] and S. Mandal[3]
[1]*Space Astronomy Group, SSIF, ISRO Satellite Centre, Bangalore, India, 560037*
[2]*Dept. of Physics, Indian Institute of Science, Bangalore, India, 560012*
[3]*Indian Institute of Space Science and Technology, Trivandrum, India, 695547*

Abstract. We use X-ray observational data from the 2011 outburst of black hole candidate IGR J17091-3624 to compute its mass. In this work we elaborate on the statistical methods used to obtain confidence intervals on our mass estimate. These methods can be used while employing similar techniques for determining mass of other black hole candidates as well.

Keywords : X-rays: individual: IGR J17091-3624 – methods: statistical

1. Introduction

We estimate the mass of black hole candidate IGR J17091-3624 in order to draw comparisons with another black hole source GRS 1915+105 and to understand the possible reasons for its observed variabilities. IGR J17091-3624 went into outburst in 2011, and underwent state transitions like other canonical black holes before displaying the complex variability features (Iyer and Nandi 2013). In order to provide proper constraints on the mass, we do a detailed study of the various sources of errors in the estimation methods, and use different statistical techniques for putting limits on the mass value. A brief description of the concepts behind the mass estimation is given in §2. §3 describes the error analysis techniques as applied to the case of IGR J17091-3624.

2. Mass Estimation methods

The best way of determining mass of binary black hole sources is to use the dynamic mass estimate obtained from radial velocity curves of the optically visible compan-

*email: `nirmal@physics.iisc.ernet.in`

ion star. In absence of such measurements, many indirect methods give estimates which are in disagreement with each other. In the test case that we consider (of IGR J17091-3624), a few indirect methods have been used, which place the mass of this source anywhere between $< 3\,M_\odot$ to $< 15\,M_\odot$ (Altamirano et al. 2011; Altamirano and Belloni 2012; Rao and Vadawale 2012; Rebusco et al. 2012). One reason for such a range of estimates is that each method has its own set of assumptions, which need to be quantified properly.

The mass estimation methods that we adopt lie under the Two Component Advective Flow (TCAF) paradigm (Chakrabarti and Titarchuk 1995). This paradigm lends three independent ways of estimating the mass : spectro-temporal correlation, the evolution of QPO frequencies and broadband spectral modelling. These methods are used when IGR J17091-3624 displays canonical black hole like states during its rising phase (Iyer and Nandi 2013). The assumptions behind these methods have been detailed in Iyer et al. (2015). Here, we detail the steps to quantify those assumptions as sources of uncertainties in the mass estimate. Such methods will also be used to quantify the mass estimates of XTE J1859+226 (Nandi et al. in preparation) and GX 339-4 (Sreehari et al. in preparation).

3. Statistical Techniques

The standard technique of interpreting error or uncertainty in a parameter is to assign it to the standard deviation of a random probability distribution function (PDF) describing the variation of the parameter. On obtaining the PDFs for mass estimate from each method, we can then combine the knowledge from these PDFs and incorporate uncertainties due to our assumptions to obtain a final set of mass limits as shown in Figure 1.

3.1 Obtaining the PDF

Bayesian based fitting methods give PDFs directly as a result of the fit routine. We use chi-square minimisation technique in all three methods. To obtain the distribution of mass from this, we use the chi-square based confidence intervals (Press et al. 1992). Confidence intervals ($C_{v_1}^{v_2}$) of a parameter (v) give the probability that the parameter estimated from the minimisation routine lies in the range of values (v_1, v_2). From the confidence intervals, we obtain area A under the PDF $p(v)$, Cumulative Distribution Function $F(v)$ and the PDF as shown by Eqn. 1-4 and left panel of Figure 1.

$$C_{v_1}^{v_2} = \int_{v_1}^{v_2} p(v)dv = P(\chi^2 > \Delta_1). \tag{1}$$

$$A_{v_1}^{v_{min}} = \frac{v_{min} - v_1}{v_2 - v_1} C_{v_1}^{v_2} \; ; \; A_{v_{min}}^{v_2} = \frac{v_2 - v_{min}}{v_2 - v_1} C_{v_1}^{v_2}. \tag{2}$$

$$F(v) = \int_{-\infty}^{v} p(v)dv = \{ {}^{A_{-\infty}^{v_{min}} - A_{v_{min}}^{v}\,,\, v<v_{min}}_{A_{-\infty}^{v_{min}} + A_{v_{min}}^{v}\,,\, v>v_{min}}. \tag{3}$$

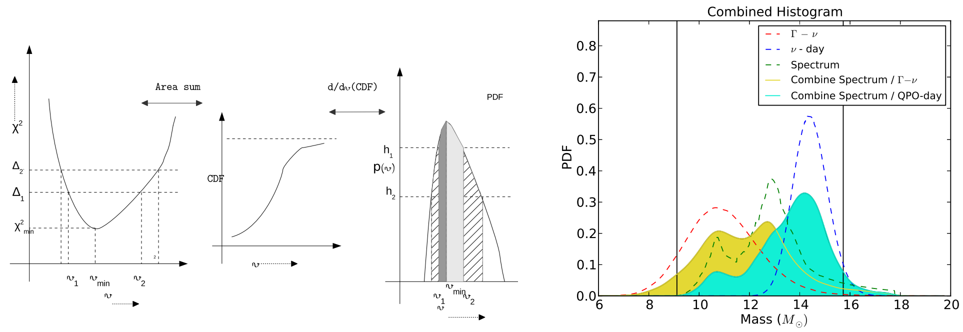

Figure 1. Left panel illustrates the steps to obtain a Probability Distribution Function (PDF) from the chi-square fit. Right panel shows the result of combining the estimates from different methods using the combined histogram. See Iyer et al. (2015) for details.

$$p(v) = \frac{d}{dv} F(v). \tag{4}$$

3.2 Combining the mass estimates

The method we employ to quantify our assumptions, is to evaluate the PDF for the same method with possible alternative assumptions. This gives us an overall PDF for a particular method, obtained by weighing each assumption properly in the overall PDF. The overall PDF is broader than before, thus indicating the increased uncertainty due to the assumptions. A detail of this weighing of various assumptions in the TCAF modelling of IGR J17091-3624 can be found in Iyer et al. (2015).

The overall PDF for mass estimates from each method can then be combined to give a final mass estimate. Multiplication of the obtained PDFs (see Eqn. 5) as indicated by the naive Bayes theorem gives a direct way of combining them. This gives us a constrained PDF which includes the results of all the estimates. We get the mass to be in the range 11.8 $M_\odot$ - 13.7 $M_\odot$ with a 90% confidence (Iyer et al. 2015). However, this method has two limitations. The first is that the PDFs must be independent of each other. Naive Bayes classifiers have been known to show a large tolerance to non independence of the PDFs (Domingos and Pazzani 1997). In parameter estimation of mass though, this could lead to a wrong estimate of the confidence levels. The second is that any unaccounted systematic offsets in one of the PDFs can either shift the final estimate or lead to an artificial reduction in the width of the combined PDF. This leads to errors in the final confidence level. To get the worst case bounds, we use the combined histogram technique, where we generate a large number (5000) of random draws from each of the PDFs and create a combined histogram of all the randomly drawn data. This is equivalent to taking a bin by bin average of the histograms representing the individual PDFs, as illustrated in Eqn. 6 and right panel of Figure 1.

Although the combined histogram itself has no particular significance, the upper and lower bounds on this histogram directly give the worst case mass limits irrespective of unaccounted dependencies or systematics. We get the mass bound from this to be in the range 9.1 $M_{\odot}$ - 15.7 $M_{\odot}$. We use these two techniques and quote the final 90% bounds (v_{low}, v_{high}) by taking the areas under the histogram which cover 5% of the PDF on either of its tail, given as $p_{combined}(v < v_{low}) = p_{combined}(v > v_{high}) = 0.05$. Additional errors or systematics can also be considered by convolving the error PDF with our final estimated PDF.

$$p_{combined}(v) = K \prod p_i(v) \qquad \text{(naive Bayes)} \tag{5}$$

$$p_{combined}(v) = K \sum p_i(v) \qquad \text{(combined histogram)} \tag{6}$$

4. Summary

We demonstrate the use of PDF based techniques to accommodate the errors in different assumptions and combine estimates from different methods. This enables a more robust evaluation of parameter values. Such methods shall also be used for mass estimation of other black hole candidates.

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